Design of Narrow Band Filters – Part 2

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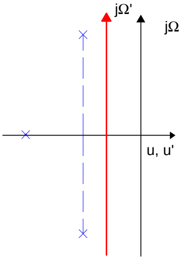
Design of Compact Filters

- Filter Design, PZ-Map
- Loss Transformation
- Modified Transfer Function
- Determination of the normalized component values of the Continued Fractions Arrangement
- Inductive and capacitive coupling
- Denormalization

Idea of the Procedure

- Design method looked at till now permits only the realization by reactive four-terminal networks. (no losses!)
- Therefore loss transformation by moving of the $j\Omega$ - axis to the left. s-data $\Rightarrow$ s'-data
- Realization of a reactive four-terminal network for s'-data.
- The circuit then carries out the actual data of the s-plane with losses.



Loss transformation 1

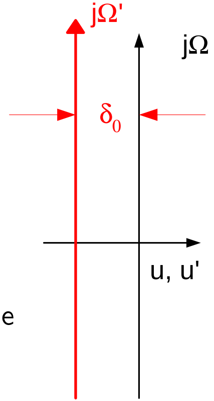
Formulation:

- All inner resonant circuits have the same quality factor and with that the same attenuation. δ_0
- The 1st circle has the attenuation $\delta 1 > \delta_0 \Rightarrow$ Consideration of the internal resistance of the source.
- The n-th circle has the attenuation $\delta_n > \delta_0 \Rightarrow$ Consideration of the input resistance of the following amplifier stage.

Loss transformation 2

$s + \delta_0 \;\Rightarrow\; s'$

$Z_e(s) \;\Rightarrow\; Z_e(s') \;\Rightarrow$ Input impedance of a reactive four-terminal network $X_e(s')$



δ_0 must be lower then the smallest absolute value of the real parts of the poles!

Continued Fractions Arrangement

With that becomes the continued fraction decomposition of the input impedance for the coupled bandpass filters:

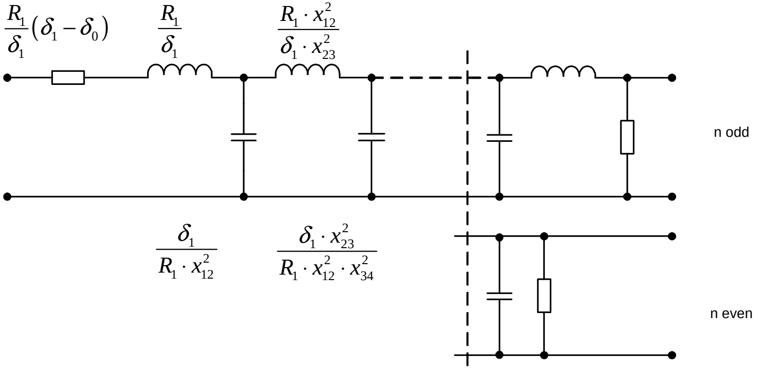
$$\frac{Z_e(s')}{R_1/\delta_1} = (\delta_1 - \delta_0) + s' + \cfrac{x_{1,2}^2}{s' + \cfrac{x_{2,3}^2}{s' + \ddots \cfrac{x_{n-1,n}^2}{(\delta_n - \delta_0) + s'}}}$$

Simplification

Simplification:

$$Z_e(s') = \frac{R_1}{\delta_1}(\delta_1 - \delta_0) + s' \cdot \frac{R_1}{\delta_1} + \cfrac{1}{s' \cdot \frac{\delta_1}{R_1 \cdot x_{1,2}^2} + \cfrac{1}{s' \cdot \frac{R_1 \cdot x_{1,2}^2}{\delta_1 \cdot x_{2,3}^2} + \cfrac{1}{s' \cdot \frac{\delta_1 \cdot x_{2,3}^2}{R_1 \cdot x_{1,2}^2 \cdot x_{3,4}^2} + \ddots}}}$$

Resulting Circuit



Remark

The input impedance results from the loss transformation for a completed equivalent LP-Reactant from the input impedance of the doubly-terminated lossy bandpass circuit.

The normalized components can be won directly by continued fraction decomposition of the fractional rational impedance function $Z_e(s)$.

It is important to say that only transfer functions without zeros can be realized by coupled resonant circuits.

Further Design Method

1. Transformation of the given BP-DTS into the normalized LP-DTS and determination of the PZ-data.
2. Loss transformation with the aim of generating the PZ'-data of the s'-plan.
3. Building the transmission function from the PZ'-data and outline of the reactive four-terminal network.
4. Continued fraction stripping down of the input impedance.
5. Denormalizing and determination of the components of the coupled bandpass filter.

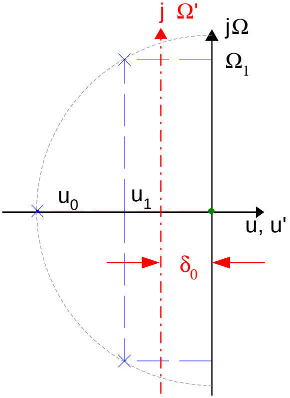
An Example shoes the further Design Method

Example: Three section bandpass filter with Butterworth approximation.

Data: $f_m = 200$ kHz
B $= 4$ kHz
$a_d = 3$ dB

PZ-Data of the normalized LP:

u_k	Ω_k
1	0
0,5	0,8660254



Loss Transformation

Loss transformation:

Condition:	$\delta_0 < \min\{\lvert u_k \rvert\}$	$= 0{,}5$
Selected:	δ_0	$= 0{,}3$

Conclusions:

$$\delta_0 = \frac{1}{Q_0 \cdot \Delta} \quad \Rightarrow \quad Q_0 = \frac{1}{\delta_0 \cdot \Delta} = \frac{f_m}{\delta_0 \cdot B}$$

$$\underline{\underline{Q_0 = 166,\overline{6}}}$$

PZ-Data

If the quality factor Q_0 (e.g. the filter coil) is predefined, then the bandwidth B cannot be chosen freely.

It is valid: $$\boxed{B \geq \frac{f_m}{\delta_0 \cdot Q_0}}$$

PZ data of the loss transformed LP:

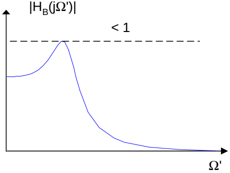
u'_k	Ω'_k
0,7	0
0,2	0,8660254

Transmission function

$$H_B(s') = \frac{K}{(s'+0,7)(s'^2+0,4s'+0,2^2+0,8660254^2)}$$
$$= \frac{K}{s'^3+1,1s'^2+1,07s'+0,553}$$
$$H_B(s') = \frac{K}{N(s')}$$

K has to be chosen so, that

$$\left|H_B(s')\right|_{s'=j\Omega'} = \left|H_B(j\Omega')\right| \quad \leq 1$$



Design of the reactive four-terminal network

$$1 - H_B(s') \cdot H_B(-s') = H_E(s') \cdot H_E(-s')$$

$$H_E(s') \cdot H_E(-s') = \frac{N(s') \cdot N(-s') - K^2}{N(s') \cdot N(-s')}$$

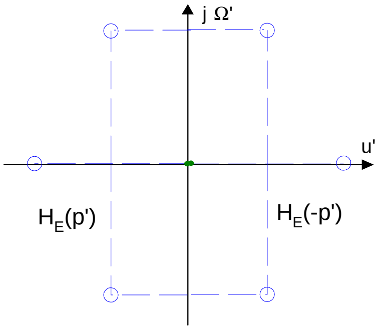
Determination of the zeros of $H_E(s') \cdot H_E(-s')$, since the poles are known.

$$H_E(s') \cdot H_E(-s') = \frac{-s'^6 - 0,93s'^4 + 0,0717s'^2 + 0,305809 - K^2}{-s'^6 - 0,93s'^4 + 0,0717s'^2 + 0,305809}$$

Zeros of $H_E(s') \cdot H_E(-s')$

Approach: $K^2 = 0,1$
Zeros of $H_E(s') \cdot H_E(-s')$: $\Longrightarrow$ numerical solution!

u'_E	Ω'_E
+0,117077	+0,829324
+0,117077	-0,829324
-0,117077	+0,829324
-0,117077	-0,829324
+0,646687	0
-0,646687	0



Remark

If K is set too greatly, so that $|H_E(j\Omega')| > 1$,the zeros of $H_E(s')$ and $H_E(-s')$ do not let themselves divide.

The zeros then lie on the imaginary axis.

Continued Fractions Arrangement of $Z_e(s)$

It gives up:

$$H_E(s') = \frac{s'^3 + 0,880841 s'^2 + 0,8529096 s' + 0,4536414}{s'^3 + 1,1 s'^2 + 1,07 s' + 0,553}$$

From this the input resistor $Z_e(s')$ is determined.

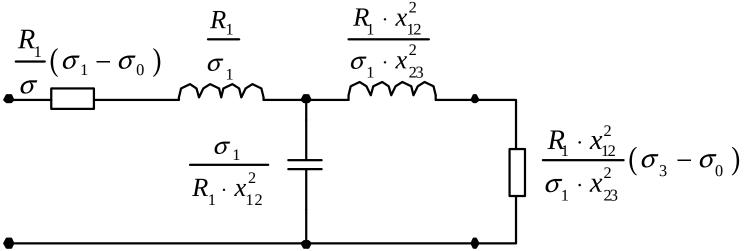
$$Z_e(s') = r_1 \cdot \frac{1 + H_E(s')}{1 - H_E(s')}$$

$$= \frac{2 \cdot s'^3 + 1,980841 s s'^2 + 1,9229096 s' + 1,0066414}{0,219159 s'^2 + 0,2170904 s' + 0,0993586} \cdot r_1$$

Continued fraction stripping down of $Z_e(s')/r_1$

$$\frac{Z_e(s')}{r_1} = 1 + 9,1257945s' + \cfrac{1}{0,2156687s' + \cfrac{1}{10,227433s' + 10,131396}}$$

From this the normalized LP gives up:



System of Equations

The system of equations arises:

$$\frac{R_1}{\delta_1}(\delta_1 - \delta_0) = r_1 \quad (1)$$

$$\frac{R_1}{\delta_1} = 9,1257945 \cdot r_1 \quad (2)$$

$$\frac{\delta_1}{R_1 \cdot x_{1,2}^2} = 0,2156687/r_1 \quad (3)$$

$$\frac{R_1 \cdot x_{1,2}^2}{\delta_1 \cdot x_{2,3}^2} = 10,227433 \cdot r_1 \quad (4)$$

$$\frac{R_1 \cdot x_{1,2}^2}{\delta_1 \cdot x_{2,3}^2}(\delta_3 - \delta_0) = 10,131396 \cdot r_1 \quad (5)$$

One gets from (**??**):

$$\frac{R_1}{\delta_1 \cdot r_1} = 9,1257945$$

This expression is contained in all other equations.

$\Longrightarrow$ Recursive solution of the system of equations.

Results

Result:

$$\begin{aligned} \delta_1 &= 0,4095795 & &> & &\delta_0! \\ x_{1,2}^2 &= 0,50809181 & &\Rightarrow & &x_{1,2} = 0,71280559 \\ x_{2,3}^2 &= 0,453363169 & &\Rightarrow & &x_{2,3} = 0,673322485 \\ \delta_3 &= 1,290609861 & &> & &\delta_0! \end{aligned}$$

Use of standard coils. $\Rightarrow$ Specification of an identical inductance for all circles.

Selected: $\underline{\underline{L = 0,1\ mH}}$

Measure of the coupling

It is valid:

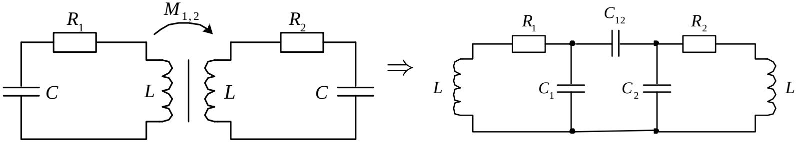
$$x_{1,2}^2 = \omega_m^2 \cdot M_{1,2}^2 \cdot \frac{\delta_1 \cdot \delta_0}{R_1 \cdot R_0} = M_{1,2}^2 \cdot \frac{\omega_m^2}{B^2 \cdot L^2} \quad \Rightarrow$$

$$M_{1,2} = \Delta \cdot L \cdot x_{1,2}$$

General: $\boxed{M_{k,k+1} = \Delta \cdot L \cdot x_{k,k+1}}$

A bandpass filter with an inductive coupling results.

Application of the capacitive coupling



$$\omega_m^2 = \frac{1}{L \cdot C}$$

$$C = C_1 + \frac{C_{12} \cdot C_2}{C_{12} + C_2}$$

As $C_{12} << C_2$ $\Rightarrow$ Approximation: $\boxed{C \approx C_1 + C_{12}}$

For the inner circles: $\Rightarrow$ Approximation: $\boxed{C \approx C_{k-1,k} + C_k + C_{k,k+1}}$

Calculation of the coupling capacities

(Philippow: Taschenbuch der Elektrotechnik, Bd. II, S. 580, Bild 7.145)

It is valid:

$$C_{k,k+1} = C \cdot \frac{\omega_m \cdot M_{k,k+1} \cdot C}{1 - \left(\omega_m^2 \cdot M_{k,k+1} \cdot C\right)^2}$$

Conversion:

$$\omega_m^2 \cdot M_{k,k+1} \cdot C = \omega_m^2 \cdot x_{k,k+1} \cdot \Delta \cdot L \cdot C = \Delta \cdot x_{k,k+1}$$

With that:

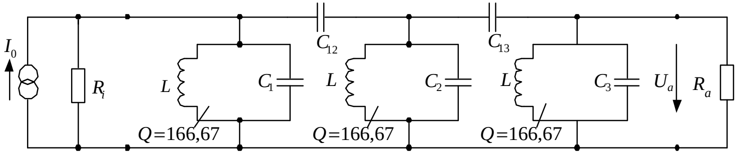
$$\boxed{C_{k,k+1} = C \cdot \frac{\Delta \cdot x_{k,k+1}}{1 - \left(\Delta \cdot x_{k,k+1}\right)^2}}$$

Denormalizing

Numerical values for the example:

$$
\begin{aligned}
C &= \frac{1}{\omega_m^2 \cdot L} &&= \underline{\underline{6,332574 \text{ nF}}} \\
C_1 &= C - C_{1,2} &&= \underline{\underline{6,2422682 \text{ nF}}} \\
C_2 &= C - C_{1,2} - C_{2,3} &&= \underline{\underline{6,1569841 \text{ nF}}} \\
C_3 &= C - C_{2,3} &&= \underline{\underline{6,2472898 \text{ nF}}} \\
C_{1,2} & &&= \underline{\underline{90,3057 \text{ pF}}} \\
C_{2,3} & &&= \underline{\underline{85,2841 \text{ pF}}}
\end{aligned}
$$

Circuit Diagram with Source and Load (C-coupling)



All circles get the same swinging Q factor. The greater attenuations for the circles 1 and 3 are realized by the input resistor of the source and the load resistor (input resistor of the following amplifier).

Required quality factor of the 1st resonant circuit: $Q_1 = 1/\Delta \cdot \delta_1$
The inner resonant circuits have a quality factor: $Q_0 > Q_1$

Calculation of source and load resistance

It is valid:

$$\left.\begin{aligned} R_{0p} &= Q_0 \cdot \omega_m \cdot L \\ R_{1p} &= Q_1 \cdot \omega_m \cdot L \end{aligned}\right\} \quad R_{1p} = R_{0p} || R_i \Rightarrow R_i = \frac{R_{0p} \cdot R_{1p}}{R_{0p} - R_{1p}}$$

$$\begin{aligned} R_i &= \frac{Q_0 \cdot \omega_m \cdot L \cdot Q_1 \cdot \omega_m \cdot L}{\omega_m \cdot L \cdot (Q_0 - Q_1)} = \omega_m \cdot L \cdot \frac{Q_0 \cdot Q_1}{Q_0 - Q_1} \\ &= \omega_m \cdot L \cdot \frac{\frac{1}{\Delta \cdot \delta_0} \cdot \frac{1}{\Delta \cdot \delta_1}}{\frac{1}{\Delta \cdot \delta_0} - \frac{1}{\Delta \cdot \delta_1}} = \frac{\omega_m \cdot L}{\Delta(\delta_1 - \delta_0)} = \underline{\underline{57,3393 \text{ k}\Omega}} \end{aligned}$$

It is valid correspondingly:

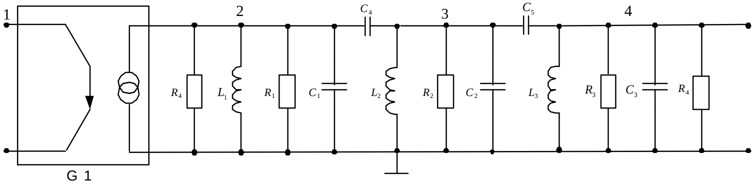
$$R_a = \frac{\omega_m \cdot L}{\Delta \cdot (\delta_3 - \delta_0)} = \underline{\underline{6,34395 \text{ k}\Omega}}$$

Testing the designed Filter

Testing the ready filter circuit with the help of a network analyzer software, e. g.

- PSpice
- Design-Center (PSpice for Windows)

Circuit Diagram for PSpice:



Electrical devices

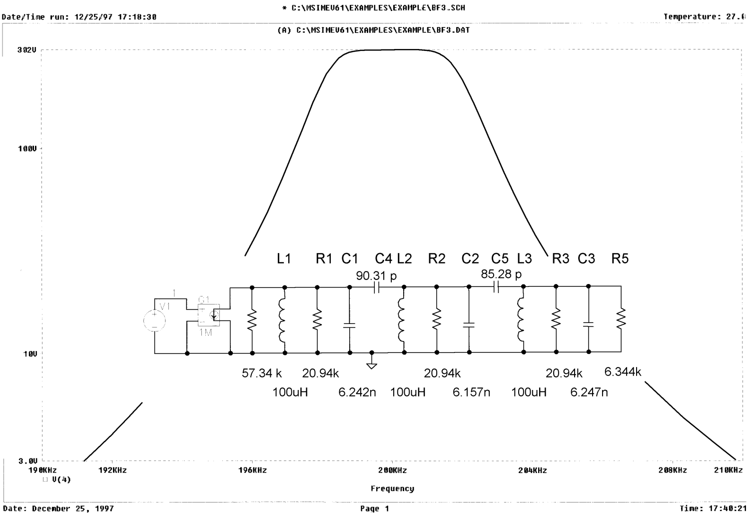
Resonant impedances of the oscillating circuits:

$$R_p = \omega_0 \cdot L \cdot Q = 2\pi \cdot 2 \cdot 10^5 \cdot 10^{-4} \cdot 166,67 = 4\pi \cdot 10 \cdot 166,67$$

$$R_p = 20,944 \text{ k}\Omega$$

$$\begin{aligned}
L_1 &= L_2 = L_3 = 0{,}1 \text{ mH} \\
R_1 &= R_2 = R_3 = 20{,}94 \text{ k}\Omega \\
R_4 &= 57{,}34 \text{ k}\Omega \\
R_5 &= 6{,}344 \text{ k}\Omega \\
C_1 &= 6{,}242 \text{ nF} \\
C_2 &= 6{,}157 \text{ nF} \\
C_3 &= 6{,}24 \text{ nF} \\
C_4 &= 90{,}21 \text{ pF} \\
C_5 &= 85{,}28 \text{ pF}
\end{aligned}$$

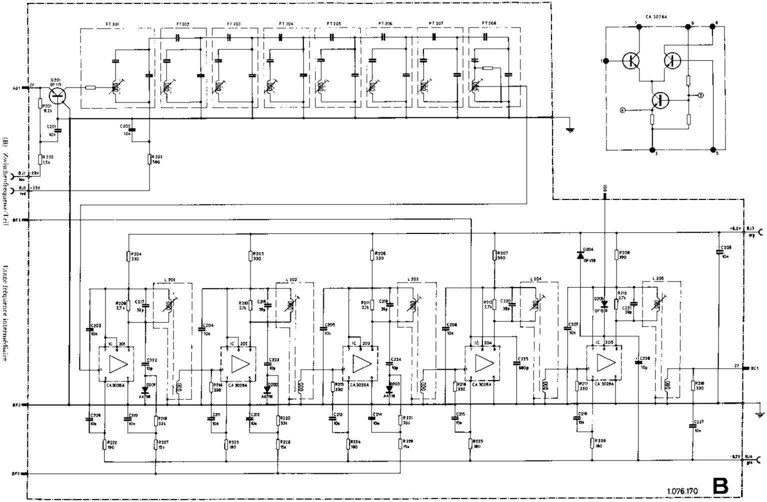
PSpice Plot


* C:\MSIMEV61\EXAMPLES\EXAMPLE\BF3.SCH
Date/Time run: 12/25/97 17:18:30
Temperature: 27.0
(A) C:\MSIMEV61\EXAMPLES\EXAMPLE\BF3.DAT
302V
100V
10V
3.0V
190KHz
192KHz
196KHz
200KHz
204KHz
208KHz
210KHz
V(4)
Frequency
L1 R1 C1 C4 L2 R2 C2 C5 L3 R3 C3 R5
90.31 p
85.28 p
V1
G1
1M
57.34 k
20.94k
20.94k
20.94k
6.344k
100uH
6.242n
100uH
6.157n
100uH
6.247n
Date: December 25, 1997
Page 1
Time: 17:40:21

Application

Application of a compact filter in the IF amplifier of the HiFi tuner ReVox A76. It becomes a 8-stage Gaussian filter with a linear phase response (constant group delay!) used.

HiFi tuner ReVox A76



Summery

- Consideration of the losses by left shift of the $j\Omega$-axis.
- With the new PZ data the transmission function of a reactive four-terminal network is calculated.
- From the continued fraction expansion of the input impedance the normalized network elements can be dimensioned.
- From the elements the couple factors and resonant circuit losses can determined.
- Calculation of the normalized devices.
- At use of customary coils the capacitive coupling of the circles is more favorable.